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TRANSITION MATRIX AND STOCHASTIC KERNEL FOR REPEATABILITY ASSESSMENT OF PERFORMANCE OF POLISH OPEN PENSION FUNDS*

Rafał Bula

University of Economics in Katowice, 1 Maja 50, 40-287 Katowice, Poland

E-mail: rafal.bula@ue.katowice.pl

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Abstract. This study focuses on the issue of repeatability and reversal of performance achieved by Polish open pension funds. For the choice of the fund investing the contributions of the future pensioner, the phenomenon of repeatability and reversal of its performance is essential. Otherwise, historical rates of return cannot be used to predict future ones, and in that case the only rational method of choosing a pension fund is a pure random selection. Contrarily to the other studies taking into account numerous variables describing open pension funds, the author analyzes the only one variable interesting from a future pensioner's viewpoint: rate of return. As a consequence the formulated conclusions are not flawed due to considering factors having no influence on future pensioners' wealth. The analysis of these phenomena was carried out by adopting the dynamics of distribution testing methods. The estimated Markovian transition matrix and the conditional density function allowed us to formulate the conclusion about the weak performance repeatability of pension funds.

Keywords: pension fund; Markov chain; performance repeatability; transition matrix; stochastic kernel

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JEL Classifications: G11, G23

1. Introduction

One of the consequences of the controversial Polish pension system reform carried out in the 1990s (sometimes called "the privatization of the pension system") (Żuk & Żuk, 2018) was the emergence of a new category of financial market participants – open pension funds (Góra, 2018), (Góra & Rutkowski, 1998). Their essential function is, above all, the investment of funds received in the form of contributions to achieve the maximum level of security and profitability of investments (Styczeń & Gomułka, 1999). Since every investment activity is subject to riskiness, it has become necessary to introduce legal solutions to protect future pensioners from taking unacceptably excessive risks and inefficient management of the investment portfolio leading to pensioners' wealth erosion (Parvi, 2014). To this end, the legislature provided the fund members with the possibility to

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change the fund and specified the minimum rate of return that the fund must achieve in order to be able to continue operating (Kominek, 2012; Kompa & Wiśniewski, 2015a). Competition between funds was to lead to the exclusion of inefficient entities, the anticipated creation of new funds was to prevent market monopolisation while legal regulations were to diminish the agency problem (Samborski, 2014). Without determining whether the described mechanism actually works (see, for example, (Brzeszczyński, Bohl, & Serwa, 2019; Chybalski, 2008, 2012; Witkowska & Kompa, 2017; Witkowska, Kompa, & Mentel, 2019)), it should be noted that for the choice of the entity investing the contributions of the future pensioner, the phenomenon of repeatability and reversal of the pension fund performance is vital. Otherwise, historical rates of return cannot be used to predict future performance, and then the only rational method of choosing a pension fund is a pure random selection. Therefore, the scientific purpose of this study is to examine the hypothesis that the performance of open pension funds operating on the Polish market is characterised by repeatability.

By repeatability of pension fund performance it is meant delivering consistently good (repeatability of success) or bad performance (repeatability of failure) in subsequent periods. There is also an opposite phenomenon to repeatability – reversal, i.e. obtaining good performance after recording bad one or bad performance after good one. The described behaviour of rates of return enables systematic achievement of better performance than others by making the proper selection of the fund or by changing it. If the phenomenon of repeatability or reversal does not occur, then forecasting the future value of accounting units is not effective and does not lead to an above average increase in future retirement benefits.

Despite significance of the raised issues not only for future pensioners, but for the whole economy, the described problem has not been thoroughly investigated. The conducted research in this field of study has been mainly devoted to the static analysis of efficiency of open pension funds. As a consequence various types of comparisons between performance of open pension funds and mutual funds, multitude of benchmark indices as well as obligatory social security fund have been presented in the literature. Unfortunately, this type of analysis is of purely static nature – it informs about past events, but the linkage between future and the past is lost. Without any additional assumptions about intertemporal relations between performance of open pension funds, the future pensioner is in reality unable to make any rational decision. It is naturally possible to assume tacitly that the future will resemble the past, but without any empirical evidence it simply becomes the classic induction problem, unsolved since it was stated by David Hume (Duignan, 2007). Alas, there are practically no empirical inquiries tackling this issue in case of Polish open pension funds. The existing research gap will be covered in this study.

There are many methods for studying the phenomenon of performance repeatability. Many of them were created in order to indirectly verify the hypothesis of strong efficiency of the capital market stating that all information (even confidential) is reflected in the prices of financial instruments (Fama, 1970). Efforts were made to prove that entities suspected of having insider information, i.e. investment and pension funds, are not able to systematically achieve better performance than the market. Although this approach is criticised by proponents of behavioural finance, it has provided many valuable tools to study the repeatability phenomenon. These include methods of distribution dynamics analysis used in this study. The novelty of the research is the application of the distribution dynamics analysis to the performance of Polish open pension funds, what has not been done before. As a result, the new and interesting results can be obtained and conclusions germane to participants of the Polish open pension funds scheme formulated.

The paper is organized as follows. Section 2 describes the capital pillar of Polish pension system – its history, investment policy of open pension funds and their performance. Section 3 includes the literature review regarding the efficiency of open pension funds. Section 4 describes theory of Markov chains and concept of stochastic kernel applied in this study as well as data analysed. Section 5 presents results obtained and discussion while the last section concludes.

2. Polish open pension funds market

The reform of the Polish pension system formally began in January 1999. The implementation of this fundamental change to then existing social security system was forced by necessity of controlling its finances in the short term as well as dramatically worsening long-term aged dependency ratio. Mere continuation of the PAYGO scheme became impossible as retirement spending in 1990-1994 increased from 8.5% to 15.6% of GDP and the “positive demographic dividend” transformed into “negative demographic dividend” (Błaszczuk, 2020). As a consequence a three-pillar system was created by adding to the obligatory PAYGO system (modified by introducing defined contribution scheme instead of defined benefit scheme) two additional capital pillars: one mandatory and the facultative third one. Connecting the amount of the future pension with individual contributions of workers during period of their labour activity was to lead the whole pension system to become self-financing in the long-term to reduce the burden to public finances.

The entities intended to manage the contributions paid in to the capital pillar of the pension system (initially 7.3% of gross salary vs. 12.2% charged by the Social Security Institution responsible for governing the first pillar fund) were the open pension funds (*otwarte fundusze emerytalne*, OFE). According to the intentions of the reform’s authors transforming workers into participants of the open pension funds market should have created incentives for managers to maximize wealth of contributors due to heavy competition between them (Bugaj, 2018). It was strongly believed that the capital pillar would outperform the mandatory PAYGO first pillar and result in significant growth of future pensions. Moreover, the predicted additional supply of funds should have led to accelerated development of domestic capital market.

From today’s perspective the history of open pension funds in Poland can be divided into three periods: the period of dynamic development (1999-2011), the transitory period (2011-2014) and the period of stagnation (2014-2020). They are analysed using such measures of development of open pension funds market like number of members, value of net assets or number of funds (Chybalski, 2005).

During the first period of existence the number of members of open pension funds rose rather steadily while the market was consolidating as number of operating pension funds fell from 20 in 1999 to 14 in 2011.

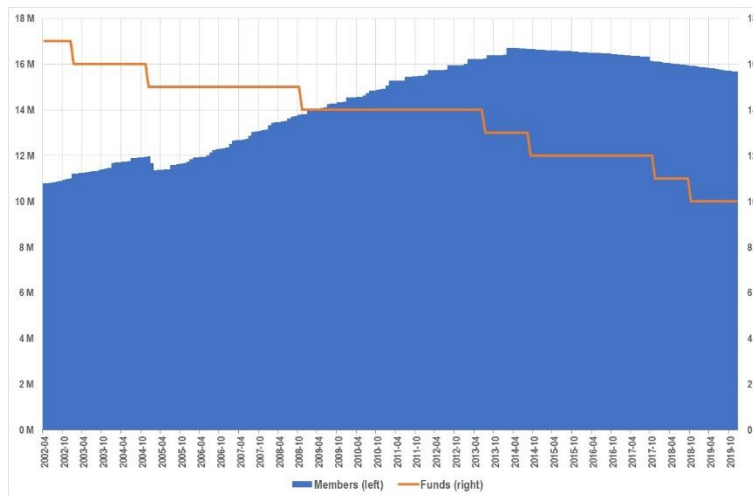


Fig. 1. Number of members of open pension funds and operating pension funds in Poland

Source: Own elaboration based on data provided by Financial Supervision Authority

The average contribution to the capital pillar of pension system remained constant at relatively high level. Consequently, at the same time a strong increase in net assets of open pension funds could have been observed (20.4% p.a.), partially due to steady inflow of contributions and partially as an effect of rising financial market. The first significant doubt about the shape of capital pillar of pension system was connected with global financial crisis 2007-2009 (Kołodziejczyk, 2019). The rapid decline of prices wiped off a substantial part of future pensioner's wealth and raised question about level of risk open pension funds participants were exposed to. The inflow of contributions was counterweighted by plummeting asset prices and as a result net assets of open pension funds declined slightly despite still increasing number of participants. Moreover, as the crisis put public finances under heavy pressure, a discussion regarding the efficiency of the existing reformed pension system and its influence on public debt became widespread.



Fig. 2. Net assets of open pension funds and their share in GDP

Source: Own elaboration based on data provided by Financial Supervision Authority

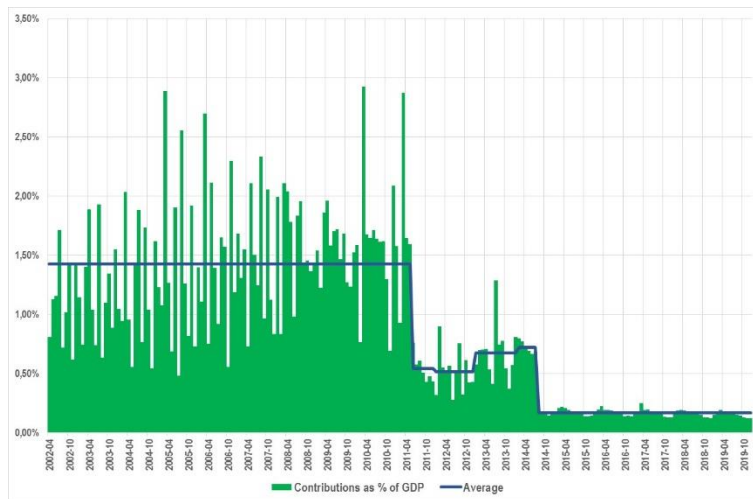


Fig. 3. Contributions paid in to open pension funds as share in GDP

Source: Own elaboration based on data provided by Financial Supervision Authority

After the crisis the combined effect of growing financial markets and increasing contributions due to the economic recovery resulted in fast growth of funds' assets and continuation of the trend in case of the number of

the members. Nevertheless, the undermined trust in capital pillar of the pension system and budget difficulties led the government to gradual reduction of the open pension funds sector (Chybalski, 2014; Gubernat, 2013; Jakubowski, 2016). Firstly, in May 2011 the contribution to the open pension funds was reduced by 68% (from 7.3% of gross salary to 2.3%) and the difference started to be transferred to Social Security Institution. Additionally, the investment limits of open pension funds were changed to allow them to gradually invest more funds in equities (we do not discuss here changes to the remaining pillars of the pension system, e.g. the raised minimum retired age, as they are not explicitly connected with functioning of open pension funds). Secondly, in February 2014 net assets of open pension funds were reduced by 51.5% due to transferring them to Social Security Institution (government bonds and government-backed securities) and at the same time funds were disallowed to invest in these types of instruments in the future. Also the obligation of investing at least 75% of assets in shares was introduced and the limits regarding investing in more risky types of instruments as well as foreign ones were liberalized. Moreover, the open pension funds participants were given the possibility of choosing once again whether they wanted to continue to contribute to the capital pillar of the system or the whole contribution should be transferred to the Social Security Institution. Only about 15.4% members decided to contribute to open pension funds (Błaszczuk, 2020). This meant radical diminution of the amount of contributions paid in to the funds (the existing assets were not withdrawn from open pension funds) despite a slight increase in the level of the individual contribution (2.92% of gross salary). Finally, the funds were obliged to gradually transfer assets of their participants starting from 10 years before reaching retirement age to the Social Security Institution. The effects of the abovementioned changes are perfectly visible on fig.2 and fig.3.

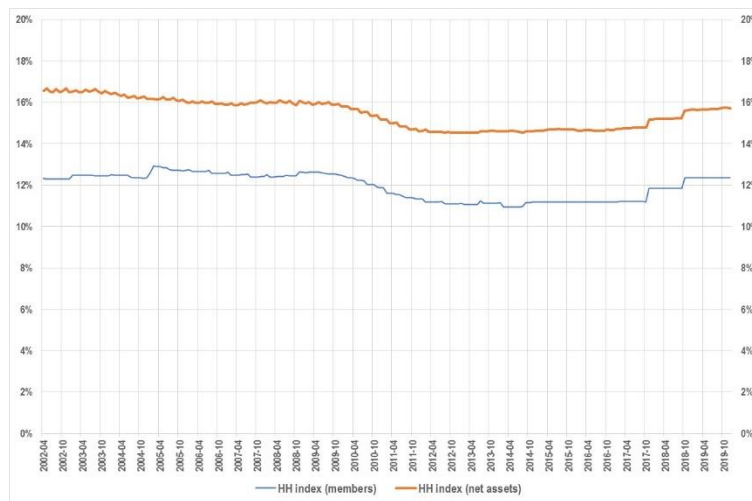


Fig. 4. Herfindahl-Hirschman index for open pension funds

Source: Own elaboration based on data provided by Financial Supervision Authority

The last period of existence of open pension funds lasting from 2014 till now is characterised by slow erosion of importance of open pension funds. The falling number of these entities (reduction from 20 by half) still manage relatively large funds (now concentrated on the stock market), but due to small group of active members (i.e. paying in contributions) and gradual withdrawal of funds of retiring participants their net assets are continuously declining. The values of Herfindahl-Hirschman index (Rhoades, 1993) show that despite this decline, the open pension funds market did not become more monopolistic (as relatively small funds disappeared from the market).

The future remains unclear. It was almost decided by the government that open pension funds would be converted into specialized open investment funds and only voluntary payments by their participants would be allowed (while still having option of transferring all pension savings to special fund administered by Social Security

Institution). The agreed time to introduce these changes was summer 2020, but due to the SARS-CoV-2 pandemic they were postponed.

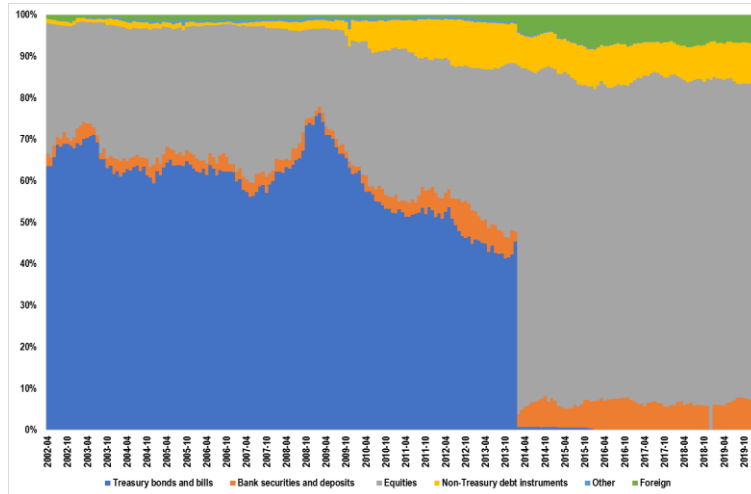


Fig. 5. Structure of investment portfolio of open pension funds

Source: Own elaboration based on data provided by Financial Supervision Authority



Fig. 6. Cumulated logarithmic rate of return of average open pension fund, benchmark and Social Security Institution

Source: Own elaboration

The decisions regulating the open pension funds market have a clear reflection in structure of portfolio held by these entities. Till February 2014 the main type of financial instruments the funds invested in were Treasury bonds and bills (63%-78%) (Bolisęga, 2013; Czerwińska, 2011; Trippner, 2012). Approximately 1/3 of assets consisted of equities. This mix of various types of assets remained relatively stable till 2014 due to only slight changes in investment limits (e.g. open pension funds were allowed to invest up to 50% of their assets in equities before 2011, 52.5% in 2011, 55% in 2012, 57.5% in 2013 and 60% in 2014). A dramatic shift in the structure of assets held by open pension funds occurred in 2014 after enacting a ban of investing in Treasury securities and transferring them to Social Security Institution. At the same time the minimum limit of equities held equal to 75% was introduced (it would decrease year by year: in 2015 to 55%, in 2016 to 35%, in 2017 to 15% and then

abolished). As a result equities became the main component of portfolios of open pension funds (approx. 85%) and despite reduction of their net assets more than by half, they remained one of the most important investors on Warsaw Stock Exchange. Moreover, this situation is quite unusual from worldwide perspective as pension funds are exposed to huge level of investment risk (Jakubowski, 2015). This solution is widely criticised by specialists as disallowing the pension funds to create freely more secure strategies (Kompa & Wiśniewski, 2014, 2015b; Kompa & Witkowska, 2015b; Lisowski, 2015).

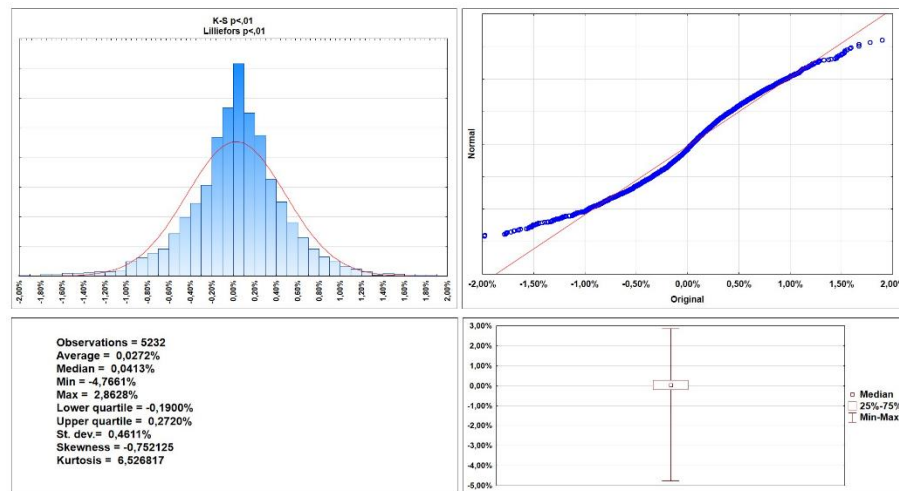


Fig. 7. Histogram, QQ-plot and box chart of daily rates of return of average open pension fund

Source: Own elaboration

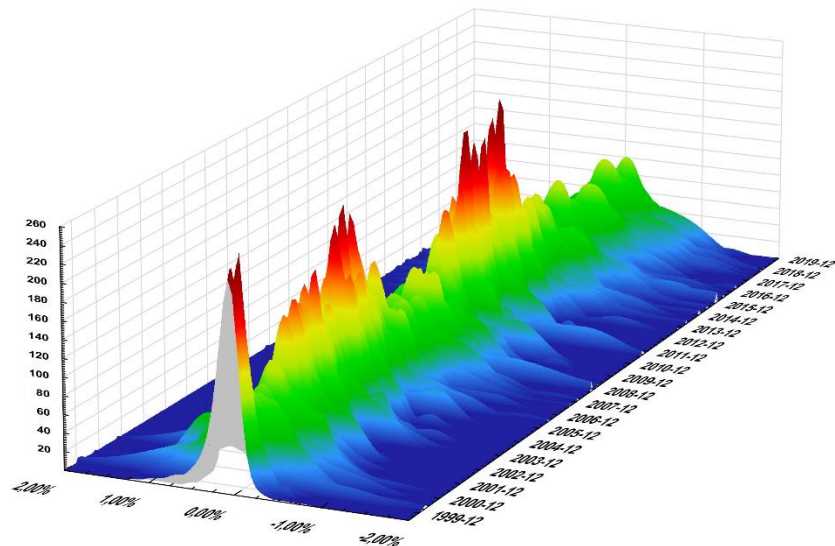


Fig. 8. Smoothed monthly distributions of daily rates of return of open pension funds

Source: Own elaboration

The imposed investment limits are perceived as a main cause of similarity between performance of open pension funds (Frasyniuk-Pietrzyk, 2008; Kompa & Witkowska, 2015a). Fig.6 shows cumulated logarithmic rates of return achieved by average fund, constructed benchmark and Social Security Institution. Before 2014 there was

no benchmark against which performance of funds could have been compared. The so called minimum rate of return was a function of average return of all funds, so it could not have been used as an independent benchmark (Jonas, 2010). As a result as benchmark was chosen a portfolio built from main Warsaw Stock Exchange index – WIG and 3M WIBOR (treated as equivalent to short-term risk-free rate) in proportions given by average portfolio of all funds. In 2014 a legal benchmark was introduced defined as portfolio built from WIG (80%) and 3M WIBOR increased by 50 bp (20%). It can be noticed that for most of the analyzed time the performance of open pension funds did not differ from benchmark while it outperformed significantly the Social Security Institution.

Statistical summary (fig.7) shows that as usually in case of financial time series the fat-tails can be observed (normality hypothesis rejected) as well as negative skewness. Fig.8 illustrates the smoothed monthly distributions of daily rates of return of open pension funds. The increase in risk exposure after 2014 is perfectly visible from the shape of the distributions drawn.

3. Literature Review

The modern analysis of performance of collective investment institutions has begun when the famous investigation by Michael Jensen was published (Jensen, 1968). Since then numerous analyses of this issue have been prepared. Unfortunately, in Poland the history of contemporary collective investment institutions is significantly shorter as these institutions could have been allowed to operate only after abolishing the centrally planned economy in early 1990s. The research devoted solely to the results achieved by pension funds began to be published after the reform of the pension system in 1999. The problem of efficiency of the newly created open pension funds became widely discussed.

The first studies analysing performance of Polish open pension funds concentrated solely on rates of return achieved by these entities (Borowski, 2004; Czechowska, 2002; Trippner, 2008). Generally it was emphasized, that the results were satisfactory from future pensioner's point of view. Unfortunately, these studies covered a very short period of time. Moreover, the authors did not take into account the problem of investment risk what makes their conclusions dubious.

The financial crisis of 2007-2009 attracted the attention of the researchers to the problem of including risk into their investigations regarding the efficiency of Polish open pension funds. Most of the studies taking into account riskiness of funds investments exploit the widely known measures connected with Capital Asset Pricing Model of Treynor, Sharpe, Lintner and Mossin and single-index model created by William Sharpe and their modifications.

(Zimny, 2011) emphasizes that risk (previously a “hidden” parameter) must be taken into account while assessing the performance of open pension funds. The presented analyses using Markowitz efficient frontier idea and Sharpe ratio confirms that open pension funds are mean-variance efficient. Moreover they are not better nor worse than benchmark index constructed from WIG index (40%) and 3M WIBOR (60%).

Witkowska et al. (Kompa & Witkowska, 2014, 2015a, 2016b, 2016a; Witkowska, 2018; Witkowska & Kompa, 2015, 2017; Witkowska et al., 2019) in a series of studies investigated the efficiency of open pension funds using a wide variety of measures (i.e. mainly risk to reward ratios). The general conclusion is that open pension funds perform better than Social Security Institution, most benchmarks and stable-growth mutual funds (i.e. mutual funds characterised by the most similar investment policies).

Table 1. Results of studies analysing efficiency of Polish open pension funds (OPF)

| Study | Period | Methods | Conclusions |
|--|-----------|--|---|
| (Czechowska, 2002) | 1999-2001 | Comparisons of rates of return | OPF achieve the aim of increasing wealth of future pensioners |
| (Borowski, 2004) | 1999-2003 | Comparisons of rates of return | OPF are highly efficient |
| (Trippner, 2008) | 1999-2007 | Comparisons of rates of return | OPF achieve the aim of increasing wealth of future pensioners |
| (Frasyniuk-Pietrzyk, 2008) | 1999-2007 | Jensen alpha | OPF are not worse nor better than benchmark |
| (Zimny, 2011) | 1999-2011 | Efficient frontier analysis, Sharpe ratio | OPF are mean-variance efficient |
| (Rusielik, 2011) | 2009 | Data Envelopment Analysis, Stochastic Frontier Analysis | Most OPF are effective |
| (Kompa & Witkowska, 2014) | 1999-2013 | Comparisons of rates of return, Sharpe ratio, Treynor ratio | OPF beat benchmark independently on market conditions |
| (Frasyniuk-Pietrzyk & Pietrzyk, 2014) | 2000-2013 | Jensen alpha, Treynor-Mazuy model, Henriksson-Merton model, Weigel model, Connor-Korajczyk model | OPF are not worse nor better than benchmark OPF selectivity abilities are poor |
| (Żebrowska-Suchodolska & Karpio, 2014) | 1999-2013 | CALMAR ratio, Omega, upside potential, Sortino ratio, Sharpe-Israelson ratio | OPF achieve results similar to performance of stable-growth mutual funds |
| (Dziwok, 2015) | 2001-2012 | Information ratio & coefficient, imperfect foresight analysis | Most OPF beat the benchmark |
| (Lisowski, 2015) | 1999-2014 | Comparisons of rates of return, Sharpe ratio, Ward's method | OPF beat benchmark, despite not fully profiting from diversification |
| (Witkowska & Kompa, 2015) | 2000-2013 | Sharpe ratio, generalized Sharpe ratio, Sortino ratio, Treynor ratio, Sharpe alpha | OPF beat Social Security Institution |
| (Kompa & Witkowska, 2015a) | 2000-2013 | Sharpe ratio, generalized Sharpe ratio, Sortino ratio, Treynor ratio, Black-Treynor ratio | OPF are not worse than Social Security Institution |
| (Kompa & Witkowska, 2016a) | 1999-2013 | Comparisons of rates of return, Sharpe ratio, Treynor ratio | OPF beat benchmarks independently on market conditions |
| (Kompa & Witkowska, 2016b) | 2009-2015 | Sharpe ratio, Treynor ratio, Jensen alpha | OPF beat stable-growth investment funds and capital market |
| (Witkowska & Kompa, 2017) | 2000-2013 | Comparisons of rates of return, Sharpe ratio, generalized Sharpe ratio, Sharpe-Israelson ratio, Sortino ratio, Treynor ratio, Sharpe alpha | OPF beat Social Security Institution |
| (Karpio & Żebrowska-Suchodolska, 2017) | 2014-2017 | Treynor-Mazuy model, Henriksson-Merton model | OPF selectivity abilities are poor |
| (Witkowska, 2018) | 2000-2013 | Comparisons of rates of return, Sharpe ratio, generalized Sharpe ratio, Sharpe-Israelson ratio, Sortino ratio, Treynor ratio, Sharpe alpha | OPF beat benchmarks, including money market portfolios and bond portfolios |
| (Wyszyński, 2018) | 1999-2018 | Comparisons of rates of return | OPF beat similar investment funds while highly dependent on market conditions |
| (Witkowska et al., 2019) | 2009-2015 | Comparisons of rates of return, Sharpe ratio, Treynor ratio, Jensen alpha | OPF beat stable-growth mutual funds; their efficiency decreases after 2014 reform |

Source: Own elaboration

These conclusions were confirmed by results of the research carried out by (Dziwok, 2015; Lisowski, 2015; Rusielik, 2011; Wyszynski, 2018) using methods as different as hierarchical clustering (Ward's method), imperfect foresight analysis or Data Envelopment Analysis and Stochastic Frontier Analysis. On the other hand there exist studies showing that abilities of managers of open pension funds are quite similar to skills of managers of mutual funds (Frasyniuk-Pietrzyk, 2008; Żebrowska-Suchodolska & Karpio, 2014). It is also noticed that their ability to select outperforming companies and instruments is rather poor, so they rather simply follow the market (Frasyniuk-Pietrzyk & Pietrzyk, 2014; Karpio & Żebrowska-Suchodolska, 2017).

Besides analysis of performance of open pension funds against an external benchmark also the issue of similarities between results of particular funds were investigated. Taking this problem into consideration from theoretical point of view, the existence of the so called minimum rate of return calculated as an internal benchmark based on the collective performance of the open pension funds induced "herd" behaviour of funds. As a consequence most studies show that independently of the performance measure adopted (Jensen alpha, Sharpe ratio, Sortino ratio, etc.) and method of analysis (hierarchical clustering, ANOVA, etc.) the results of open pension funds are hardly distinguishable (Frasyniuk-Pietrzyk, 2008; Kompa & Witkowska, 2015a; Lisowski, 2015; Marcinkiewicz, 2015; Mikołajczak & Bajak, 2017). There are also studies claiming lack of homogeneity of the analysed entities, but it can be observed that these conclusions are based on multicriteria techniques taking into account also net assets, number of members of funds, etc. (Franków, 2015; Obidziński, 2008). Due to significant differences between characteristics other than rates of return employing multicriteria techniques inevitably leads to misleading conclusions of heterogeneity while from the future pensioner's viewpoint the only important variable is achieved rate of return.

The abovementioned studies concentrate on analysis which might be called static as it does not take into account possible (and quite probable) changes in variables describing particular entities in time. The importance of stability of results achieved by particular open pension funds for future pensioners is obvious. The main result are summarized in tab.2.

Table 2. Results of studies analysing changes in characteristics of Polish open pension funds (OPF)

| Study | Period | Methods | Conclusions |
|--------------------------|--------------------------|--|---|
| (Miszczynska, 2003) | 2001-2002 | PROMETHEE, AHP | Performance of OPF is stable over time |
| (Miszczynska, 2004) | 2004-2005 (forecasts) | PROMETHEE, AHP | Performance of OPF is stable over time |
| (Mikulec, 2008) | 2002-2007 | Taxonomic measures, Walesiak measure, Kendall's tau | Performance of OPF is stable over time |
| (Bialek & Mikulec, 2009) | 1999-2007 | Comparisons of rates of return, R/S analysis, Hurst exponent | Performance of most OPF does not exhibit long-memory effect |
| (Włodarczyk, 2010) | 2005-2009 | Taxonomic measures, Spearman's rho, Kendall's tau | Performance of OPF is unstable over time |
| (Kucharski, 2011) | 2009-2010 | PROMETHEE | Performance of OPF is rather stable over time with spectacular exceptions |
| (Buła, 2014) | 2007-2011 | Taxonomic measures, Spearman's rho | Performance of OPF is stable over time |
| (Bukietyńska, 2017) | 2002-2010 | Sharpe ratio, analysis of inversions | Performance of OPF is unpredictable and random over time |

Source: Own elaboration

Most studies provide arguments in favour of the hypothesis that results of open pensions funds are characterized by time stability (Buła, 2014; Kucharski, 2011; Mikulec, 2008; Miszczynska, 2003, 2004). What must be remembered while formulating conclusions based on this empirical evidence is the method used by the authors.

They do not concentrate solely on rates of return, but exploit multicriterial analysis investigating also e.g. net assets, number of the members, level of fees etc. The common practice in this case is to consider multiple criteria while calculating appropriate taxonomic measures and most of the abovementioned variables are stable over time. The persistence of the resulting measure is a simple consequence of the normative assumption that performance of open pension funds is measured by taking into account a variety of different variables. But from the point of view of a single pensioner the only significant characteristic is rate of return. Quite contrarily to the previous results, in this case almost the pure randomness can be observed over time (Białek & Mikulec, 2009; Bukietyńska, 2017; Włodarczyk, 2010). An additional confirmation of these results are conclusions drawn by (Chybalski, 2011; Jędrzychowska, 2011). According to their analyses of connections between funds performance and members transfers there does not exist any dependence between rates of return of particular fund and inflow of members to it. Assuming rationality of *homo oeconomicus* this result is not surprising.

Unfortunately, the number of studies analysing repeatability of rates of return solely is rather small and they regard only the first years of functioning capital pillar of Polish pension system. There exists an evident scientific gap, which is covered in the following paragraphs of this study.

4. Data and Methodology

Apart from the selection of the research method, it is also extremely important to choose the right measure of pension fund performance. The use of measures that take into account only the profitability of investments leads to their distortion, because it omits the level of risk borne. Therefore, this study adopted the Sharpe ratio (Sharpe, 1966) and Modigliani risk-adjusted performance (Modigliani & Modigliani, 1997) as they are free of this deficiency. The analyses used daily valuations of accounting units of open pension funds made available by the money.pl. The Sharpe ratio and Modigliani–Modigliani measure were calculated for 82 quarter periods starting from the third quarter of 1999 using the formulas (Bacon, 2013):

$$S_{i,t} = \frac{\bar{r}_{i,t} - \bar{r}_{f,t}}{s_{i,t}}, \quad (1)$$

and:

$$M_{i,t}^2 = \frac{\bar{r}_{i,t} - \bar{r}_{f,t}}{s_{i,t}} s_{m,t} + \bar{r}_{f,t}, \quad (2)$$

where $S_{i,t}$ – the revised Sharpe ratio of the i -th pension fund in period t , $\bar{r}_{i,t}$ – the average daily logarithmic return from the accounting unit of the i -th fund in period t , $s_{i,t}$ – the standard deviation of the daily logarithmic excess return from the accounting unit of the i -th fund in period t , $\bar{r}_{f,t}$ – the average daily logarithmic return from the risk-free asset in period t , $s_{m,t}$ – the standard deviation of the daily logarithmic excess return from the market in period t , i – the pension fund number ($i = 1, 2, 3, \dots, 17$), t – the period number (a quarter; $t = 1, 2, 3, \dots, 82$; $t = 1$ corresponds to the third quarter of 1999). The quarters in which the fund started or ended operations were not considered. The WIBID TN rate was adopted as the estimated risk-free interest rate, while the WIG index was chosen as a market proxy (data provided by stooq.com). The obtained Sharpe ratio and M^2 values were then used to examine the performance repeatability of open pension funds using the methods described in the following part of the paragraph.

Provided that $\{X_t : t \in \mathbb{N}\}$, where \mathbb{N} – the set of natural numbers, is a stochastic process with a finite state space (for convenience, let us assume that the values, which the process can take, belong to the set $\{x_1, x_2, \dots, x_n\}$). We can say that X_t is a Markov chain, if the probability of the process adopting a certain value at time t depends only

on the value that the process assumed at time $t-1$, and not on the values previously adopted. If the probability does not additionally depend on time, then we call such the process a homogeneous Markov process. Then the probability of being at the moment t in the state j provided that it was previously in the state i ($i, j \in \{1, 2, 3, \dots, n\}$) is constant, it is marked as $p_{i,j}$. Considering $p_{i,j}$ for all permissible pairs (i, j) , one can build a square matrix with n rows showing all transition probabilities for period:

$$Q(1) = \begin{bmatrix} p_{1,1} & p_{1,2} & \dots & p_{1,n} \\ p_{2,1} & p_{2,2} & \dots & p_{2,n} \\ \dots & \dots & \dots & \dots \\ p_{n,1} & p_{n,2} & \dots & p_{n,n} \end{bmatrix}. \quad (3)$$

The element at the intersection of the i -th row and the j -th column is equal to $p_{i,j}$. If we now denote by $\bar{m}_t = [p_1^t, p_2^t, \dots, p_n^t]$ the vector the probabilities with which the process X_t can take the appropriate values ($\Pi(\{\omega \in \Omega : X_t(\omega) = x_k\}) = p_k^t$), then $\bar{m}_{t+1} = \bar{m}_t \cdot Q(1)$. Similarly, transition probabilities are considered over s periods and a transition matrix appropriate for s periods is defined as $\bar{m}_{t+s} = \bar{m}_t \cdot Q(s)$ (obviously $Q(s) = [Q(1)]^s$). Moreover, additional information about the process can be obtained by examining the vector \bar{m}_∞ illustrating the probability of being in particular states after an infinite number of transitions ($\bar{m}_\infty = \lim_{t \rightarrow \infty} \bar{m}_t$) regardless of the initial state \bar{m}_0 (it has been proven that \bar{m}_∞ always exists) (Epstein, Howlett, & Schulze, 2003).

Assuming that the stochastic process reflecting the value of the Sharpe ratio and M^2 in subsequent quarters in relation to the performance of open pension funds is a homogeneous Markov process with a transition matrix identical for all funds, it is possible to estimate individual transition probabilities. To simplify, the number of states can be assumed $n = 2$, while being in state 1 is equivalent to achieving a result below the median, and state 2 in other cases. We then estimate $p_{1,1}$ as the ratio of the number of funds that failed in two consecutive periods to the number of funds that failed in the first period; $p_{1,2}$ as the ratio of the number of funds that failed in the first period and met with success in the second period to the number of funds that failed in the first period, etc. Actually, the transition probabilities should be estimated for each two successive periods separately. However, a small number of operating open pension funds would suggest making calculations in relation to observations accumulated in all periods, which in practice means assuming that the process is close to reaching a stationary status. In the absence of repeatability or reversal, the calculated quotients should be close to $1/2$. The significance of deviations from this value can be tested using χ^2 test. The test statistic is:

$$\chi^2 = \frac{(N_{1,1} - N/4)^2 + (N_{1,2} - N/4)^2 + (N_{2,1} - N/4)^2 + (N_{2,2} - N/4)^2}{N/4}, \quad (4)$$

where N – the total number of observations, $N_{1,1}$ – the number of funds that in two consecutive periods achieved a result below the median, etc. In the absence of a correlation between performance results, this statistic has an asymptotic distribution χ^2 with one degree of freedom. If this hypothesis is rejected then it should be considered that the performance of pension funds is characterised by repeatability or reversal (conclusions regarding the direction of dependence are formulated by analysing the system of transition probabilities) (Jackowicz & Filip, 2009).

The method using the transition matrix, however simple, has a significant disadvantage – it forces discretization of the state space of the process. The decision on the number of states and the method of assigning the observation to a specific state is made arbitrarily, which can significantly affect the performance achieved. Therefore, this study also uses a method based on estimation of the stochastic kernel.

In the previous part of the paragraph, the stochastic process was analysed which had a finite state space. This reasoning can be extended to cases when the state space is an infinite set (e.g. a set of real numbers). Then the use of the concept of transition matrix is impossible. In this situation, the concept of stochastic kernel was introduced. The stochastic kernel is the function of two variables (point and set), which for a given point is a probability distribution relative to a given set and meets certain weak regularity conditions (Feller, 1971). They can be treated as a form of a transition matrix with an infinite number of rows and columns. For the stochastic process $\{X_t : t \in \mathbb{N}\}$ with the set of states, which are the subset of real numbers, the stochastic kernel can be determined by the density function $f(x_{t+1}|x_t)$ of a conditional distribution of a random variable $(X_{t+1}|X_t)$ (Jackowicz & Filip, 2009). If we assume that X_t is a homogeneous Markov process, then the density function $f(x_{t+1}|x_t)$ will be independent of t . Its estimation is based on the use of equation $f(x_{t+1}|x_t) = \frac{g(x_t, x_{t+1})}{h(x_t)}$, where $g(x_t, x_{t+1})$ is the density of the two-dimensional random variable (X_t, X_{t+1}) , whereas $h(x_t)$ is the random variable density X_t . The estimation of the conditional density function will take the form $\hat{f}(x_{t+1}|x_t) = \frac{\hat{g}(x_t, x_{t+1})}{\hat{h}(x_t)}$, where $\hat{g}(x_t, x_{t+1})$ is an estimator of the random variable density function (X_t, X_{t+1}) , and $\hat{h}(x_t)$ is an estimator of the random variable density function X_t . The first step is therefore to estimate the function $g(x_t, x_{t+1})$ and $h(x_t)$. Kernel estimators are used to this end. With N observations of a random variable (X_t, X_{t+1}) (i.e. pairs of (x_{it}, x_{it+1}) , $i = 1, 2, 3, \dots, N$) we can specify the estimator of function $g(x_t, x_{t+1})$ as:

$$\hat{g}(x_t, x_{t+1}) = \frac{1}{Nab} \sum_{i=1}^{i=N} K_2 \left(\frac{x_t - x_{it}}{a}, \frac{x_{t+1} - x_{it+1}}{b} \right), \quad (5)$$

and of function $h(x_t)$:

$$\hat{h}(x_t) = \frac{1}{Na} \sum_{i=1}^{i=N} K_1 \left(\frac{x_t - x_{it}}{a} \right), \quad (6)$$

where $K_2(x, y)$ and $K_1(x)$ are functions that satisfy the conditions $\int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} K_2(x, y) dx dy = 1$ and $\int_{-\infty}^{+\infty} K_1(x) dx = 1$, whereas

a and b – parameters (so-called bandwidths). In this study, it was assumed that $K_1(x)$ is a density function of de Moivre-Gauss standard distribution and $K_2(x, y)$ (in accordance with the recommendations in the paper by Hyndman et al.) meets the condition $K_2(x, y) = K_1(x)K_1(y)$ (Hyndman, Bashtannyk, & Grunwald, 1996). Parameters a and b were estimated using the formulas (Silverman, 1986):

$$a = \left(\frac{4}{3N} \right)^{\frac{1}{5}} s_{x_t}, \quad (7)$$

$$b = \left(\frac{4}{3N} \right)^{\frac{1}{5}} s_{x_{t+1}}, \quad (8)$$

where s_{x_t} and $s_{x_{t+1}}$ are the standard deviations from the sample calculated for the random variable X_t and X_{t+1} , respectively (Gajek & Kałuszka, 1994). Due to the small number of open pension funds operating on the Polish market, the estimations were made for the observations collected in all periods. As in the case of the transition

probabilities, the estimation of the conditional density function $f(x_{t+1}|x_t)$ should be carried out for each two successive periods separately. The use of observations from all periods prompted by the amount of data is, in practical terms, synonymous with the assumption that process X_t is strictly stationary. Then the density functions $g(x_t, x_{t+1})$ and $h(x_t)$ do not change over time. The approach adopted in this study was used, inter alia, in the previously mentioned work of Jackowicz and Filip (Jackowicz & Filip, 2009).

Performing the estimation results in obtaining a two-dimensional conditional density function whose full graphical presentation is only possible in three-dimensional space. It is, however, much easier to analyse the orthogonal projection of the examined surface. To do this, one should plot isohypses connecting the points with the same value of the conditional density function. Interpretation of the shape of the stochastic kernel then takes place on a similar principle as examining the terrain with the help of a hypsometric map. In this study, it was assumed that the abscissa reflects the process values from moment t , and on the ordinate from the moment $t+1$. A sample plot is presented below.

The key to assessing the performance repeatability is the location of the stochastic kernel relative to the straight line with the equation $x_{t+1} = x_t$ (inclined at an angle of 45° , marked with a white dashed line in the figure). The position of the kernel along it proves the lack of mobility – funds that have achieved good (bad) performance in the current period will also most likely record good (bad) performance.

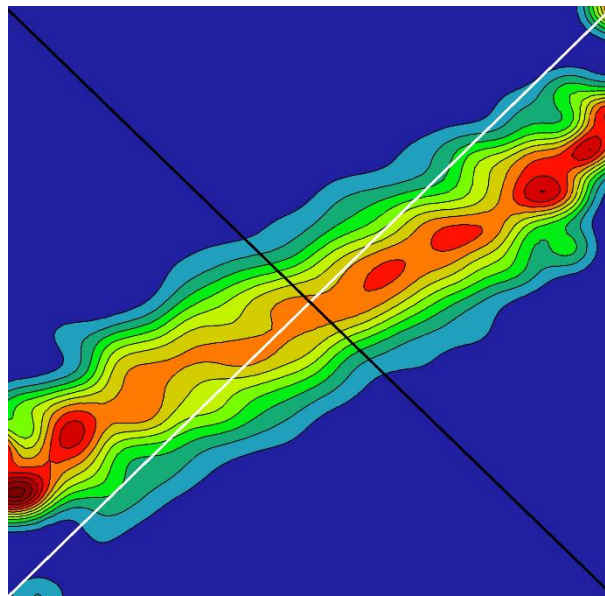


Fig. 9. Sample orthogonal projection of stochastic kernel

Source: Own elaboration

Counterclockwise rotation indicates that the differences between funds are widening over time. Clockwise rotation towards straight line $x_{t+1} = 0$ proves the disappearance of differences and the decrease in the impact of the performance of the current period on the performance of future periods. The phenomenon of reversal could be said when the kernel is arranged approximately along a straight line inclined at an angle of 135° (black dashed line in the figure).

5. Results and Discussion

Using the data made available by the money.pl and stooq.com in accordance with the guidelines contained in previous paragraph, Sharpe ratios and Modigliani risk-adjusted performance for individual open pension funds were calculated. The charts show their average value and typical area of variation for each quarter.

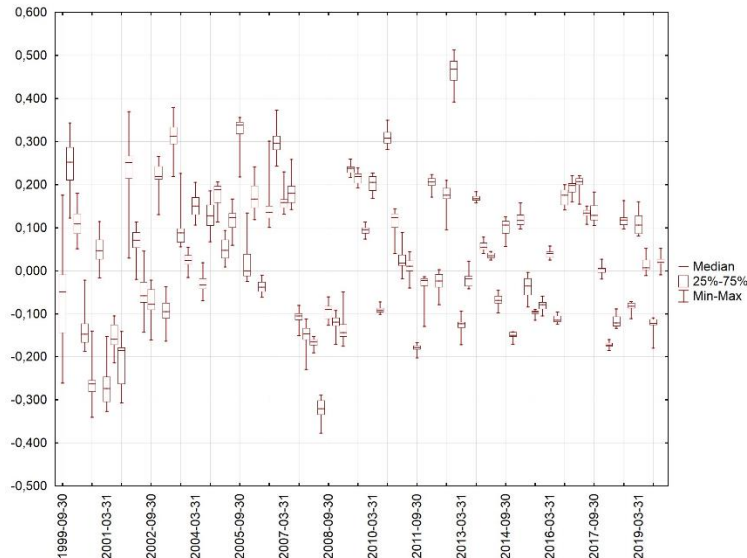


Fig. 10. Average value and typical area of Sharpe ratio volatility for open pension funds in the period of Q3 1999 – Q4 2019

Source: Own elaboration (data provided by stooq.com and money.pl)

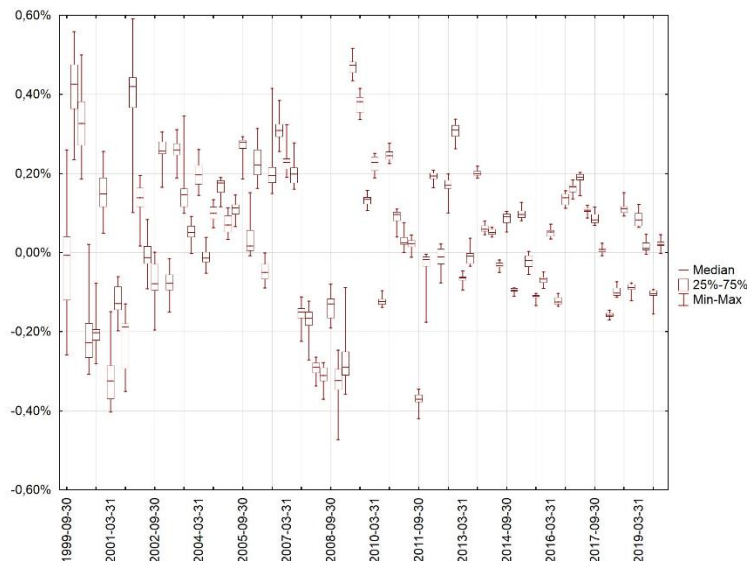


Fig. 11. Average value and typical area of Modigliani risk-adjusted performance volatility for open pension funds in the period of Q3 1999 – Q4 2019

Source: Own elaboration (data provided by stooq.com and money.pl)

The plots presented show a clear trend of the performance of pension funds to be similar over the period considered. In the next step, a two-line transition matrix was estimated, with the discretization using the sample median equal to $Me = 0.032$ (Sharpe ratio):

$$Q(1) = \begin{bmatrix} 0.527 & 0.473 \\ 0.466 & 0.534 \end{bmatrix}$$

or $Me = 0.05\%$ (Modigliani-Modigliani measure):

$$Q(1) = \begin{bmatrix} 0.530 & 0.470 \\ 0.465 & 0.535 \end{bmatrix}.$$

Then, by making 2^{100} iterations, an approximation of the invariant vector was obtained in case of Sharpe ratio:

$$\bar{m}_{\infty} \approx [0.496 \quad 0.504]$$

and Modigliani-Modigliani measure:

$$\bar{m}_{\infty} \approx [0.497 \quad 0.503].$$

In addition, the χ^2 test statistics were calculated. The p-value for Sharpe ratio is 0.036 while for the M^2 0.026. The obtained results indicate the statistically significant, though rather weak repeatability (resulting from the analysis of estimated transition probabilities). In the long run, the initial choice of the fund does not seem to affect the amount of the pension – only systematic monitoring of performance achieved and a change of fund (if it achieved an unsatisfactory rate of return) could increase the amount of the future benefit.

To obtain confirmation of the above results, an analysis using a stochastic kernel was also carried out. Based on the estimated unconditional densities, the conditional density function $f(x_{t+1}|x_t)$ was calculated and is shown on the graphs. To simplify the analysis, charts of an orthogonal projection of a stochastic kernel on a plane were also plotted. The projected shapes of the kernel allow us to confirm the previously formulated conclusions.

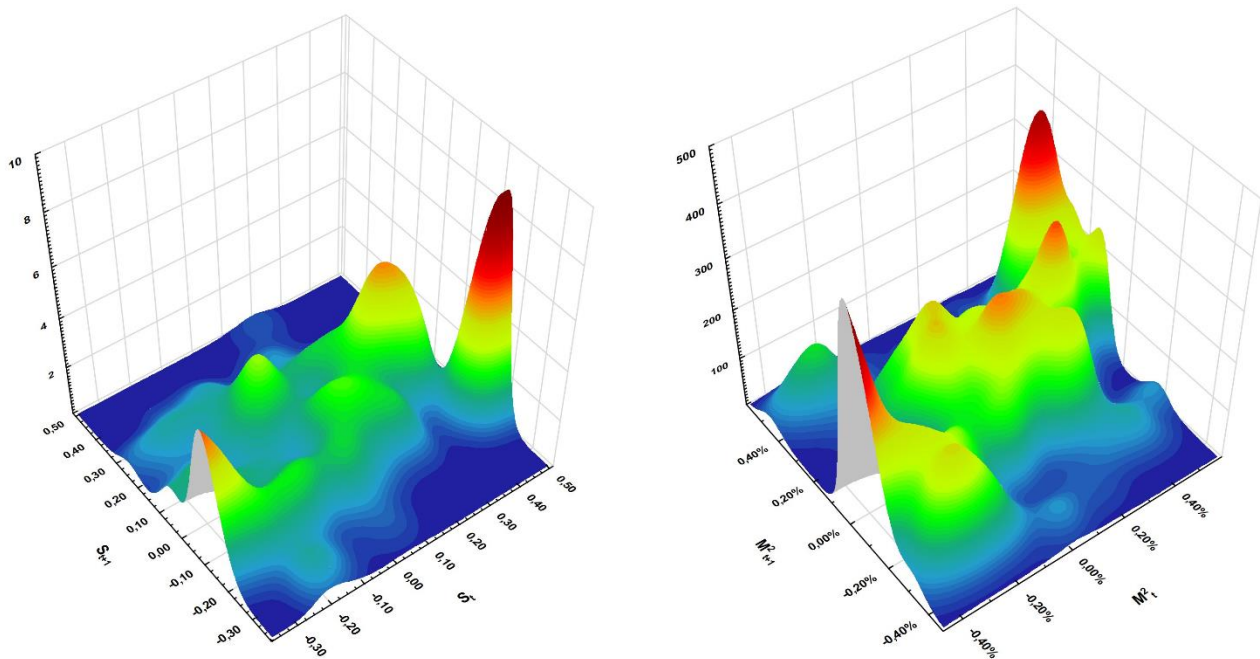


Fig. 12. Estimated conditional density function for Sharpe ratio (left graph) and M^2 (right graph)

Source: Own elaboration (data provided by stooq.com and money.pl)

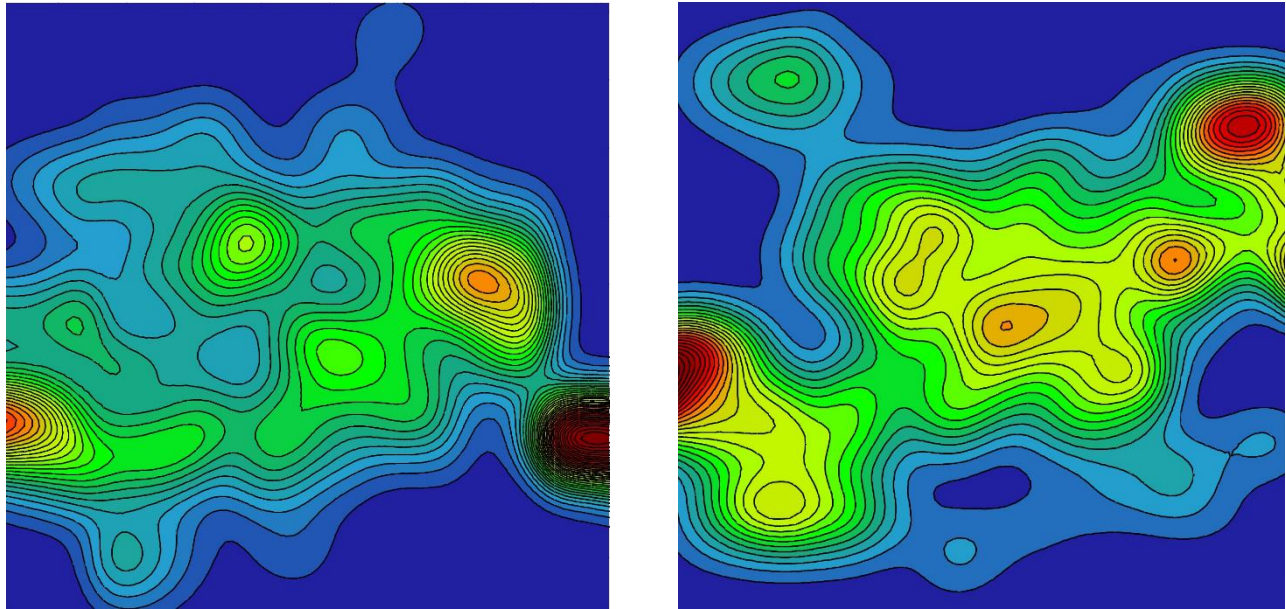


Fig. 13. Orthogonal projection of the estimated stochastic kernel for Sharpe ratio (left graph) and M^2 (right graph)

Source: Own elaboration (data provided by stooq.com and money.pl)

They indicate some performance repeatability, in particular for the central values. On the other hand, the trend of performance convergence of open pension funds is much more marked for the extreme values, especially in case of Sharpe ratio.

The results presented for quarterly data remain valid while analysing Sharpe ratio and Modigliani-Modigliani measure for yearly periods. The transition matrices are almost the same and only minor differences might be observed in case of shape of stochastic kernel.

The results obtained using theory of Markov chains and estimated stochastic kernel are consistent with the conclusions formulated by (Bukietyńska, 2017) using test of inversion of Sharpe ratio for open pension funds. Moreover, the author's observations cover 20-year period while (Bukietyńska, 2017) only 9-year period. The presented results remain in agreement with assumption of future pensioner's rationality and empirical evidence provided by (Jędrzychowska, 2011). Thus, we may conclude, that the initial choice and potential changes of open pension fund in the future are not important for future pensioner's wealth. As a consequence, we may treat the set of open pension funds as unity or at least a very homogenous one.

Conclusions

The calculations made allowed us to draw the conclusion about the phenomenon of very weak, though statistically significant, performance repeatability achieved by open pension funds operating on the Polish market. The analysis of empirical data also confirms previous local and international results (Bohl, Lischewski, & Voronkova, 2011; Draženović, Hodžić, & Maradin, 2019; Kurach, 2019) indicating that the rates of return recorded by individual entities become increasingly similar over time. The study highlighted the clear advantages of the methods adopted to study distribution dynamics – above all, the ease of interpretation of the results obtained. These methods are a valuable tool in the process of analysing various economic phenomena.

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Rafał BUŁA Ph.D. is assistant professor at Department of Investment, College of Finance, University of Economics in Katowice, Poland. His dissertation devoted to fractal analysis of financial markets has been awarded by the Chair of Polish Financial Supervision Authority as the best thesis defended in 2016 in field of finance. Research interests: investments, financial markets, risk assessment, fractal analysis.
ORCID ID: orcid.org/0000-0002-9890-7078

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